1. What is an equivalence relation? What are some real-world examples of equivalence relations?

2. Determine whether the following relations are equivalence relations.

   (a) The relation $\mathcal{R}$ on $\mathbb{Z}$ given by
   $$\mathcal{R} = \{(a,b) \mid |a-b| \leq 2\}.$$

   (b) The relation $\mathcal{R}$ on $\mathbb{R}^2$ given by
   $$\mathcal{R} = \{(a,b) \mid ||a|| = ||b||\}$$
   where $||a||$ denotes the distance from a to the origin in $\mathbb{R}^2$

   (c) Let $S = \{a,b,c,d\}$. Let $\mathcal{R}$ be the relation on S given by
   $$\mathcal{R} = \{(a,a),(b,b),(c,c),(a,b),(b,c),(a,c),(b,a),(c,b),(c,a)\}.$$

3. How many ways can $A = \{1,2,3,4\}$ be partitioned? How about $A \cup \{5\}$? Develop an algorithm to compute the number of possible partitions when $|A| = n$.

4. Find the amount of ways to partition $\{1,2,3,4,5,6,7,8,9,10,11,12\}$ into two partitions $\mathcal{A}$ and $\mathcal{B}$ such that $|\mathcal{A}| \notin \mathcal{A}$ and $|\mathcal{B}| \notin \mathcal{B}$.

5. Prove the following combinatorial identities:

   (a) $\sum_{k=0}^{n} \binom{n}{k} = 2^n$
   (b) $\sum_{i=1}^{n} i = \binom{n+1}{2}$
   (c) $\binom{n}{2}\binom{n-2}{k-2} = \binom{n}{k}\binom{k}{2}$
   (d) For any n and k such that $n \geq k$, prove that $\sum_{i=k}^{n} \binom{i}{k} = \binom{n+1}{k+1}$
   (e) $\sum_{i=0}^{n} 2^k \binom{n}{k} = 3^n$
   (f) $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$

6. Calculate the coefficients of the following polynomials terms

   (a) $(2x+4)^7$ for x^5
   (b) $(x+2y+5)^6$ for xy^2
   (c) $(x^2+3x-2)^9$ for x^3
   (d) $\prod_{i=1}^{10}(x+i)$ for x^8

7. In which row of Pascal's Triangle do three consecutive entries occur that are in the ratio $3:4:5$?

8. A positive integer is called ascending if there are at least two digits and each digit is less than any digit to its right. How many ascending positive integers are there?

9. Given an unlimited supply of unit equilateral triangles of six different colors, how many equilateral triangles with side length 2 can you make that are unique with respect to rotations and reflections? (Think Zelda Triforce shape).

10. A mother purchases 5 blue plates, 2 red plates, 2 green plates, and 1 orange plate. How many ways are there for her to arrange these plates for dinner around her circular table if she doesn't want the 2 green plates to be adjacent?

11. You are placed on the origin of the Cartesian plane and can only travel on grid lines.

    (a) If you take 10 moves, how many ways can you return to the origin?

(b) If you take 15 moves, how many ways can you end up at (2,1)?

12. Suppose you are placed on the origin of a 2-dimensional grid. If you can only move up and to the right, how many possible paths are there to get to the point $(5, 3)$?

13. You just arrived in New York City at Grand Central. By only walking south on avenues and west on streets, how many ways can you get to Madison Square Garden? (Map below)

   (a) Assume there are 5 avenues to cross and 11 streets to cross with all streets and avenues unobstructed and continuous.

   (b) (Challenging!) However, there are two streets that are cut off: 41st street at Bryant Park and 32nd street at Madison Square Garden. With this in mind, how many ways are there now?