

1. Classify the following relations (reflexive, symmetric,...):

(a) $R = \{(a, b) \mid a = b\}$

(b) $R = \{(a, b) \mid a < b\}$

(c) $R = \{(a, b) \mid a \mid b\}$

(d) $R = \{(a, b) \mid \gcd(a, b) = 1\}$

(e) $R = \{(a, b) \mid 3 \mid (a - b)\}$

2. What is an equivalence relation? What are some real-world examples of equivalence relations?

3. Determine whether the following relations are equivalence relations.

(a) The relation \mathcal{R} on \mathbb{Z} given by

$$\mathcal{R} = \{(a, b) \mid |a - b| \leq 2\}.$$

(b) The relation \mathcal{R} on \mathbb{R}^2 given by

$$\mathcal{R} = \{(a, b) \mid \|a\| = \|b\|\}$$

where $\|a\|$ denotes the distance from a to the origin in \mathbb{R}^2

(c) Let $S = \{a, b, c, d\}$. Let \mathcal{R} be the relation on S given by

$$\mathcal{R} = \{(a, a), (b, b), (c, c), (a, b), (b, c), (a, c), (b, a), (c, b), (c, a)\}.$$

4. Prove the following combinatorial identities:

(a)

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

(b)

$$\sum_{i=1}^n i = \binom{n+1}{2}$$

(c) For any n and k such that $n \geq k$,

$$\sum_{i=k}^n \binom{i}{k} = \binom{n+1}{k+1}$$

5. Calculate the coefficients of the following polynomials terms

(a) $(2x + 4)^7$ for x^5

(b) $(x^2 + 3x - 2)^9$ for x^3

(c) $(x + 2y + 5)^6$ for xy^2

(d) $\prod_{i=1}^{10} (x + i)$ for x^8