PILOT

Discrete Math
Problem-Set #2
Spring 2019

1. Find the number of anagrams of the word 'PILOT'.

2. License plates generally have 6 characters consisting of capital letters and numbers. Find the amount of license plates with the following constraints.

   (a) License plates with 2 numbers.

   (b) License plates such that no 2 numbers are next to each other

3. A local breakfast bar has 3 options: bagels, omlettes, and pancakes. For one week starting on Sunday, Charlie eats there every morning. Assume Charlie eats only one of the items on a given day, i.e., Charlie will not eat bagels and pancakes in one day.

   (a) Find how many different ways Charlie can eat breakfast throughout the week.

   (b) Find the amount of possibilities if Charlie doesn't want to eat the same breakfast two days in a row.

   (c) Find the amount of possibilities with (b) and if Charlie wants to eat pancakes on Saturday (end of the week).

4. There are 3 distinct urns, and 5 balls. Compute the amount of ways to place the 5 balls in the 3 urns given the following scenarios.

   (a) All 5 balls are unique.

   (b) All 5 balls are identical.

   (c) All 5 balls are identical and no urn is empty.

5. Jessica teaches a weekend lab class with 3 JHU students and 3 UMD students. Each student needs a lab partner.

   (a) Find the amount of possible ways the 6 students can form partnerships if each JHU student's partner is from UMD.

   (b) How about when any student can pair up with any other student?

6. How many subsets of $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ are there such that the sum of their elements is 10? How about if $0 \in A$ as well?

7. Let $A = \{\varnothing, 1\}$, $B = \{2, 3, 4\}$, and $C = \{4, 5, 6\}$. Find the cardinalities of the following sets: (a) A; (b) $B \cup C$; (c) $B \cap C$; (d) $\{A, B, C, 2\}$; (e) $C \setminus B$; (f) 2^A (g) 2^{2^A}.

8. A certain high school requires students to take an English and Math exam. 90% of the students passed the English exam, and 80% passed the Math exam, while 75% passed both exams. If the total number of students who failed both exams is 20, find the total number of students who took the exam.

9. Prove or give a counterexample to the following claims:

   (a) For any two sets A, B, $2^{A \cup B} = 2^A \cup 2^B$.

   (b) For any two sets A, B, $2^{A \cap B} = 2^A \cap 2^B$.

PILOT Learning - Tip of the Week

Did you know that coffee refills are discounted at most places around campus including Charmar, Brody Café, and Levering Café? Save your cup, save some cash and save the planet.