PILOT

Discrete Math
Problem-Set #11
Spring 2019

1. A graph has n vertices:

   (a) What is the maximum amount of edges that it can have?

   (b) If the graph is complete, how many edges does it at least have?

   (c) Find the minimum amount of edges to guarantee it to be a connected graph.

2. If a graph has 10 identical vertices, how many ways can we connect the vertices such that all edges have a degree of 2?

3. Is adjacency reflexive, symmetric, and/or transitive?

   (a) If a graph has 10 unique vertices, how many edge configurations allow adjacency to be transitive?

4. Prove that among any 6 people, three of them are mutual acquaintances or mutual strangers. Assume that each relationship is an acquaintance relationship or a stranger relationship.

5. Prove that a graph must have two vertices of the same degree.

6. For which n can you draw the edges of the complete graph K_n without taking your pencil off the page?

7. How many non-isomorphic graphs on 5 vertices have a clique/independent set size of 3?

8. What is the difference between a maximum and maximal clique or independent set?

9. Assume that graph G is connected with 8 identical vertices. How many possible graphs are there such that there are 3 cut edges?

10. Let $G = (V, E)$ be a graph with $V = \{1, 2, 3, 4, 5, 6\}$. In Figure 1, we show the graph G with vertex i removed for $i = 1, 2, \ldots, 6$ (e.g. $G - 2$ is the graph obtained after removing vertex 2 from G).
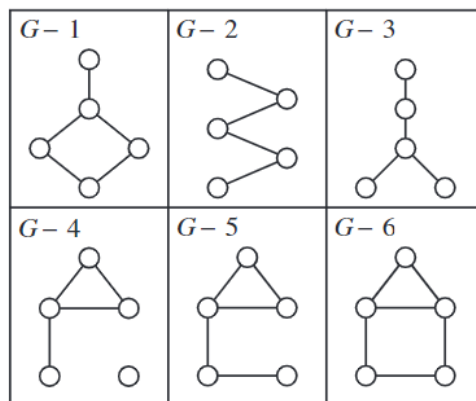

Figure 1:

Find the graph G.

11. (Challenging) Suppose there is a mouse with a $3 \times 3 \times 3$ cube of cheese. The mouse starts by eating a $1 \times 1 \times 1$ corner of the cube. The mouse would like to eat each individual $1 \times 1 \times 1$ cube, but can only travel to an adjacent cube once it has consumed one. Is it possible for the mouse to get to the core of the $3 \times 3 \times 3$ cube where the core is the final piece eaten by the mouse?

PILOT Learning - Tip of the Week

Did you know that you can charge your laptops/phones in the lockers in Brody? Helpful to know when finals preparation coming up!