



**Discrete Math  
Problem-Set #10  
Spring 2019**

1. Find the totient of integers from 1 to 20.
2. Find  $\varphi(80)$ ,  $\varphi(97)$ ,  $\varphi(132)$ .

We want to find an efficient method to compute the totient of any integer. The following problems will give us insight into the formula:

3. For a prime number  $p$ , find  $\varphi(p)$ . Why?
4. Find  $\varphi(2^{10})$ . How about  $\varphi(2^n)$ ? How about  $\varphi(p^n)$  for any prime number  $p$ ?
5. Given that  $\varphi(x) = y$ , find  $\varphi(x^2)$ . How about  $\varphi(x^n)$ ?
6. For any two unique prime integers  $p$  and  $q$ , prove that  $\varphi(pq) = \varphi(p)\varphi(q)$ . How about  $\varphi(p^x q^y)$ ?
7. Any number's prime factorization can be written as  $n = p_1^{e_1} p_2^{e_2} \dots p_n^{e_n}$ . Find  $\varphi(p_1^{e_1} p_2^{e_2} \dots p_n^{e_n})$ .

Additional totient function problems.

8. Find all integers  $n$  such that  $\varphi(n) = 12$ .
9. Prove that  $\varphi(n) = 14$  has no solution.
10. Find all integers  $n$  such that  $\varphi(n) = \varphi(2n)$ .

RSA Encryption

11. What do Alice and Bob do in RSA encryption?
12. Prove that the encoding and decoding functions successfully decrypt Alice's message.
13. Given that the primes used are  $p = 11$ ,  $q = 37$ , and that Bob chose  $d = 29$ , find  $e$  and Alice's encrypted message if her original message is  $M = 48$ .

***PILOT Learning - Tip of the Week***

Have you ever suffered from procrastination? Take a look at the top ten list of ways to stop procrastinating and get ready for finals.

Top Ten List to Stop Procrastination

1. Break it down! Look at your work and break it into smaller steps in order to simplify it.
2. Change your environment – find a new place to study.
3. Create a detailed timeline with specific deadlines.
4. Eliminate your procrastination pit-stops – disable email and social media notifications.
5. Hang out with people who inspire you to take action.
6. Find a group to study with.
7. Tell others about your goals – it makes you more accountable to achieve them.
8. Seek out someone who has already achieved the outcome you want and ask their advice.
9. Take a much needed break and give your brain a rest! A little bit of rest time can help you strengthen your focus.
10. Stop over-complicating things – there is never a perfect time to start studying or a perfect pen/notebook to take notes - just take action and start!