PILOT

Discrete Mathematics
Problem-Set #1
Spring 2019

1. Construct truth tables for the following statements:

   a) $Q \vee \neg P \implies \neg R$

   b) $(A \wedge B) \iff (\neg C \vee D)$

2. Choose one of $(\impliedby, \implies, \iff)$ for each pair of statements:

$$\begin{array}{rcl} A \text{ is a square} & (\impliedby, \implies, \iff) & A \text{ is a rectangle} \\ B \text{ is divisible by 5 and 2} & (\impliedby, \implies, \iff) & B \text{ is divisible by 10} \\ C * D \text{ is odd} & (\impliedby, \implies, \iff) & C \text{ and } D \text{ are both odd} \end{array}$$

3. Determine if the following statements are true. If they are, prove them; if not, give a counterexample.

   a) It is possible to express the logical AND operator ($\wedge$) using only OR ($\vee$) and NOT ($\neg$).

   b) It is possible to express the logical OR operator ($\vee$) using only AND ($\wedge$) and NOT ($\neg$).

4. Divisibility rules:

   a) Prove that if the last digit of an integer n is divisible by 2, then n is divisible by 2.

   b) Prove that if the last 2 digits of an integer n is divisible by 4, then n is divisible by 4.

   c) What is the divisibility rule of 8? Prove it.

   d) What is the pattern of divisibility rule for powers of 2? Why?

5. Measures of central tendency:

   a) What is the arithmetic mean? What is the geometric mean? Write their formulas for 2 numbers.

   b) When is one mean greater than the other mean? When are they equal? Assume you're only working with non-negative real numbers.

   c) Prove your statement in part (b).

6. Factoring of positive integers:

   a) Develop an algorithm or method to find the amount of factors of an integer.

   b) Develop an algorithm or method to find the prime factorization of an integer.

   c) Given the prime factorization of an integer, find how many factors it has.

7. How many factors of 10^{99} are not factors of 10^{88}?

8. There are 1000 lockers in a high school for 1000 students numbered from 1 to 1000. All lockers start out closed. Student 1 opens all lockers. Student 2 closes all lockers that are multiples of 2. Student 3 opens or closes all lockers that are multiples of 3, and so on. After the 1000th student opens or closes all lockers that are multiples of 1000, how many lockers will be open?

9. Prove that no triangular number is a prime number.

10. Prove that for all $n \in \mathbb{Z}$, $5n^2 + 3n + 1$ is odd.
11. Prove that for all $n \in \mathbb{Z}$, $30 \mid n^5 - n$.
12. Prove or disprove that $3^n + 2$ is prime for all positive integers n.
13. Prove or disprove that if $a^2 = b^2$, then $a = b$ for real numbers a and b.

PILOT Learning – Tip of the Week

The Arts & Sciences Office of Academic Advising and the Career Center are hosting a February extravaganza event: "Success Cafe" on Monday, February 11, 2019 from 4:00-5:30pm in the Glass Pavilion. Its goal is to promote how academics, campus resources, and experiential activities all contribute to student success. There will be lots of campus offices hosting games and distributing prizes, come learn about what they can offer you!